N-BODY DYNAMICS LEADING TO GW SOURCES

Ladislav Šubr
with
Myank Singhal, Jaroslav Haas, Pavel Kroupa,
Georgios Loukes-Gerakopoulos, Vladimír Karas and others. . .

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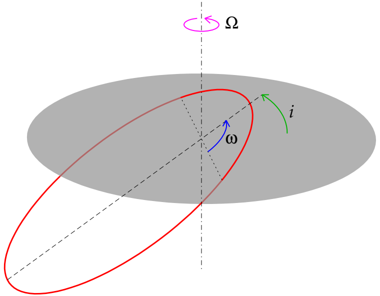
N-body dynamics and gravitational waves

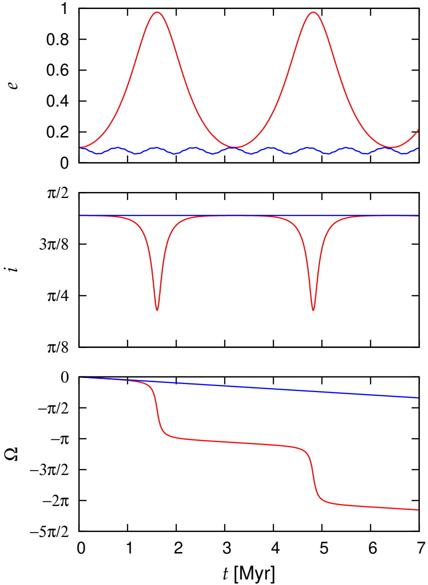
Various points of view:

- physics - GR vs. Newtonian dynamics
- GW signal - information about processes in N-body environments
- influence of N-body system on individual inspirals (signal)
- influence of GR/GW on evolution of N-body systems
- tools

Kozai-Lidov oscillations



- shape of the orbit: a, e
- orientation of its plane: i, Ω
- orientation within the plane: ω
- position on the orbit: ν

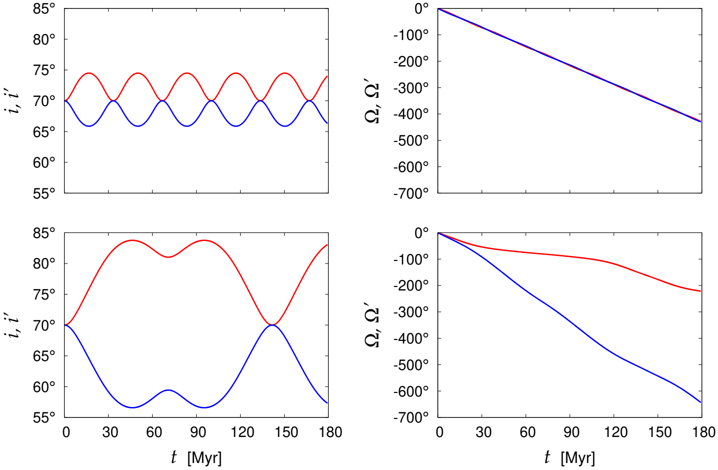


Kozai-Lidov oscillations

- already the PN1 correction leads to damping of K-L oscillations
- extreme eccentricities / extremely small pericentre passages still possible, yet with smaller rates in comparison to pure Newtonian dynamics (e.g., Karas & Šubr 2007)
- pattern of K-L cycles not likely to be detectable in the final inspiral / GW signal
- at some moment, K-L cycles may accelerate shrinkage of compact body orbits, i.e., effectively increase rates of inspirals
- K-L oscillations also suggested as solution to the 'final parsec' problem, i.e., being a driver towards SMBH-SMBH coalescence

VHS – specific setup of 4-body dynamics

(Haas, Šubr & Vokrouhlický, 2011)



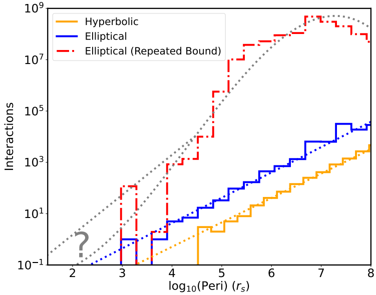
Close encounters of compact stellar remnants

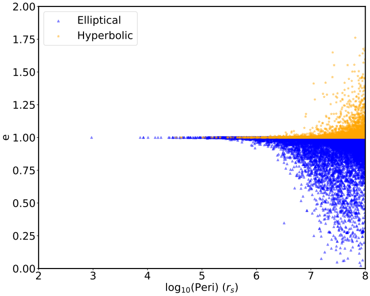
Mukherjee, Mitra, Chatterjee, 2021, MNRAS, 508: "Gravitational wave observatories may be able to detect hyperbolic encounters of black holes"

Singhal & Šubr: N-body modeling seem to be necessary to get realistic estimates of rates and properties of close encounters

Close encounters of compact stellar remnants

- 600 integrations of self-gravitating star cluster
- 20 000 stars, including 37 black holes and 219 neutron stars
- $\approx 10^{-14}$ of hyperbolic and $\approx 10^{-11}$ of elliptic encounters with pericentre within 3-10 R_{Schw} per cluster per year





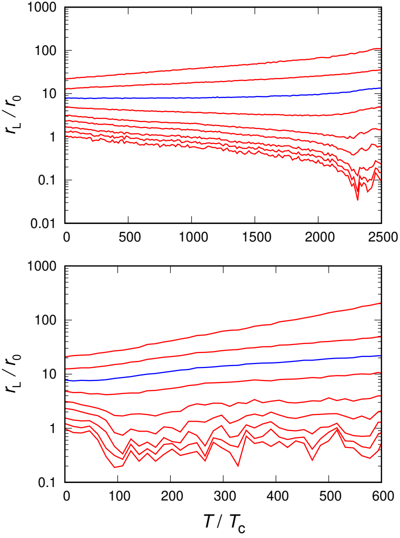
Collapse of relativistic clusters

Kroupa, Šubr, Jeřábková & Wang, 2020, MNRAS, 498:
"Very high redshift quasars and the rapid emergence of supermassive black holes"

- assume a hyper-massive ($M_c \sim 10^8\, M_\odot$) star cluster
- zero metallicity + high density → top-heavy mass function
- rapid stellar evolution → cluster of stellar mass black holes
- if gravitational radiation surpasses binary heating → cluster collapses, formig a SMBH seed

Collapse of relativistic clusters

- self-gravitating systems tend to core collapse
- binary heating prevents the "gravothermal catastrophe"
- gravothermal oscillations and slow decay through three-body interactions



Collapse of relativistic clusters

a bit of formulae:

hard/soft binary boundary: $a_{h/s} \approx \dfrac{GM_{BH}}{\sigma^2}$

binary-single collision rate: $t_{coll} \approx \dfrac{\sigma}{8\pi G \rho_{BH}\, a}$

GW decay time-scale: $t_{GW} \approx \dfrac{a^4\, c^5}{128\, G^3 M_{BH}^3}$

$$t_{coll} > t_{GW} \quad \Rightarrow \quad \sigma^{11} > \sigma_{crit}^{11} \approx \frac{1}{4} \rho_{BH} G^3 M_{BH}^2 c^5$$

$$t_{relax} \approx 0.01 \frac{\sigma^3}{G^2 M_{BH}\, \rho_{BH}}$$

$$\approx 100 \left(\frac{M_{BH}}{10 M_\odot} \right)^{-5/11} \left(\frac{\rho_{BH}}{10^8 M_\odot/\mathrm{pc}^{-3}} \right)^{-8/11} \mathrm{Myr}$$

Summary

- K-L oscillations not so effective in strong gravity, but still may contribute to inspiral rates and perhaps influence their initial properties
- ‘VHS’ - 4-body dynamics may lead to ‘coherent’ or ‘assisted’ inspirals
- rates of close encounters of compact stellar remnants in star clusters are maybe much smaller than expected by Mukherjee et al., but more effort from N-body modellers is needed to make robust conclusions; motivation from the GW community will be welcome
- formation of SMBHs through collapse of relativistic BH clusters – a lot of work may be done on our side, but insight from GW community would be beneficial at the current stage of the model already